STOXX® Index-Based Risk-Controlled Portable Smart Beta Strategies

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Abstract

A large number of smart beta indices share the characteristics of allocating weight to constituents not on the basis of market capitalization, but on the basis of other criteria, which can be related to risk, to fundamentals or simply to number of constituents.

These indices have become increasingly popular over the last few years, since it has been noticed that in the long term they all seem to have the potential to outperform their market-capitalization-weighted counterparts, thanks to exposure to alternative equity risk factors that are well documented in the financial literature, such as size and value. The long-term outperformance potential of indices whose weighting scheme is unrelated to market-cap can also be mathematically explained within the framework of stochastic portfolio theory (SPT).

This research paper investigates the possibility of building a "portable smart beta" program based on a properly chosen selection of smart beta indices. Analogously to a "portable alpha" program, where the objective is to extract skill ("alpha" in finance community jargon) from a portfolio of actively managed investment strategies, in a "portable smart beta" program we aim to extract a combination of alternative risk premiums from a set of chosen smart beta indices.

We show that, subject to the definition of a volatility-control policy and of a simple timing algorithm, the performance differential between a properly defined portfolio of STOXX smart beta indices and the corresponding portfolio of cap-weighted indices may provide a very interesting investment opportunity, offering high standards of transparency and liquidity as well as low implementation costs.

We believe that the portfolio of STOXX smart beta indices we introduce in this note could find quite a wide range of uses:

1) It could be employed as a long-only investment solution aiming to beat the corresponding benchmark of market-cap-weighted indices.
2) The strategy, which takes long exposure to the portfolio of STOXX smart beta indices and short exposure to the corresponding portfolio of cap-weighted indices, could be used to enhance the return of cash or of any predefined benchmark by exploiting the notion of "portable performance."

1. Introduction and literature review

Over the last few years, smart beta indices have gained popularity by claiming to offer risk-adjusted performance superior to that of traditional market-cap-weighted indices. While there exist smart beta equity indices that select stocks based on predefined characteristics (for instance, high dividend yield or low volatility), but then weight them based on their market capitalization, in this research paper we focus on those smart beta indices whose allocation methodology deviates from market-cap weighting.

Three main typologies of rule-based, non-cap-weighted allocation approaches have been proposed in the literature. In risk-focused approaches, portfolio weightings are only a function of specific risk properties of the constituents. A first example of this risk-focused approach is given by the minimum variance portfolio, first suggested by Haugen and Baker [1991], which arises naturally as the left-most portfolio on Markowitz's efficient frontier and in simple terms can be thought to be the fully invested portfolio with minimum risk.

A second example of the risk-focused approach is represented by the risk parity portfolio, first introduced by Qian [2005, 2006], whose properties have been extensively studied by Maillard *et al*. [2010]. It is defined as the portfolio in which the risk contribution from each asset is made equal on an ex-ante basis, maximizing risk diversification.

A simplified variant of the risk parity portfolio, which is mathematically obtained by assuming an equal-correlation matrix (i.e., a correlation matrix in which all the off-diagonal elements are supposed to be equal to each other and equal to the average asset universe correlation), is the inverse-volatility portfolio, in which assets are weighted according to the inverse of their volatility.

STOXX provides risk-based smart beta indices for different geographies. STOXX constructs a world minimum-variance index, defined in the STOXX® Global 1800 Index, and then applies the same methodology to different regions (Europe, North America and Asia/Pacific) and single countries, including the United States, Japan, Canada and Australia, both in a constrained and an unconstrained version. The constrained version aims to optimize the portfolio with respect to variance, while not modifying too much the other attributes (country/industry exposure) relative to the reference cap-weighted benchmark. The unconstrained version provides a strategy index that is minimized for volatility. At the same time, it is optimized with minimal constraints, which pertain to tradability and investability, and it is not restricted to follow a specific benchmark too closely.

In addition, STOXX provides for Europe a selection of low-risk-weighted indices, representing the lowest volatility companies within the STOXX® Europe 600 Index universe. Components are selected based on a twelve-month historical volatility ranking. Only the K lowest volatility companies are included in the low-risk-weighted indices, under the assumption that the stocks that have been the least volatile for the past twelve months will continue to record below-average volatility for the subsequent quarter (until the subsequent rebalancing). Weightings are then calculated by using the inverse of the twelve-month historical volatility.

The second typology of rule-based, non-cap-weighted allocation approaches is represented by the agnostic equally weighted portfolio, in which there is no apparent link between any market or risk-related information and portfolio weightings. An equally weighted portfolio allocates a fraction 1/N of the portfolio to each of the N assets available for investment. DeMiguel, Garlappi and Uppal [2009] evaluate the out-of-sample performance of the equally weighted strategy relative to the portfolio policy defined by the sample-based mean-variance portfolio model (and also that of some of its extensions, designed to reduce the impact of estimation error). They conclude that no mean-variance portfolio model is consistently better than the equally weighted strategy in terms of Sharpe ratio, certainty-equivalent return or turnover. Out of sample, the gain from mean-variance optimization appears to be more than offset by estimation errors in expected returns and covariances. Several equally weighted indices are available in the market from different index providers, including STOXX.

The third and last typology of rule-based, non-cap-weighted allocation approaches is represented by fundamentals-focused approaches, in which portfolio weightings are a function of some fundamentals of the constituents: for instance, the expected rate of return that is due to cash flows (such as coupons or dividends). A popular example of the fundamentals-focused allocation approach is the high-dividend-yield portfolio. In their seminal paper on fundamental indexation, Arnott, Hsu and Moore [2005] rank all companies by trailing five-year average gross dividends, select the top one thousand companies under this metric, and include each of these companies in the high-dividend fundamental index at its relative metric weighting. They show that a fundamental index weighted by gross dividends substantially outperforms a cap-weighted index.

Concerning fundamentals-weighted indices, STOXX provides a set of maximum dividend indices (reviewed quarterly), where the selection rule is based on the requirement that companies have a dividend payment within the upcoming quarter, and dividend data are based on company announcements and estimations made a few months in advance. STOXX also provides a set of select dividend indices (reviewed annually), where only dividend-paying companies with a nonnegative historical five-year dividend-per-share (DPS) growth rate and a defined dividend to earnings-per-share (EPS) payout ratio are selected. Index components are sorted based on the reported annualized net dividend yield relative to the net dividend yield of the corresponding home market. Both the maximum dividend and the select dividend indices are dividend-weighted.

Different authors have come up with empirical studies analyzing whether any of the above rule-based, non-cap-weighted allocation approaches can be considered superior from a return-versus-risk perspective in comparison with cap-weighted indices. Chow, Hsu, Kalesnik and Little [2011] find that most rule-based, non-cap-weighted allocation strategies outperform their cap-weighted counterparts because of exposure to value and size factors. Leote, Lu and Moulin [2012] compare different rule-based, non-cap-weighted allocation strategies on an equity universe (equally weighted portfolio, two variants of risk parity portfolios, the minimum variance portfolio and the maximum diversification portfolio) and analyze the factors behind their risk and performance. They show that each of these strategies, irrespective of its underlying complexity, can be explained by a few equity style factors: low beta, small cap, value and low-residual volatility. Gander, Leveau and Pfiffner [2013] show that investing in just one type of rule-based, non-cap-weighted allocation methodology often leads to unwanted concentration and cluster risks. They claim that in order to avoid this problem, it is crucial to diversify across the different rule-based, non-cap-weighted allocation methods.

Oderda [2015] addresses the question of why a particular rule-based, non-cap-weighted allocation approach or a combination thereof should emerge as a superior portfolio construction methodology from a return-versus-risk perspective, relative to a market-cap-weighted benchmark. He relies on the SPT framework of Fernholz [2002] to study the evolution of portfolio wealth relative to a market index. SPT is a descriptive theory of financial markets that is capable of accounting for the impact of compounding and rebalancing on portfolio returns—too often neglected in the literature. It is defined on a closed universe of companies and assumes that asset logarithmic returns follow a Brownian motion. Most importantly, it relies on the notion of market diversity, according to which no single asset can dominate the market in finite time. This constraint is quite intuitive and even implicit in anti-trust legislation. Since the market portfolio holds all assets, including the largest cap, diversity implies the largest market-cap asset cannot remain the largest growth asset indefinitely.

Starting from the SPT decomposition of the logarithmic return of a generic portfolio relative to the corresponding reference market index, it is possible to show that in the resulting return decomposition formula two terms can be identified: a drift term, which an investor should aim to maximize, and a noise term, which is stochastic but can be shown to remain bounded if the market remains diverse all the time.

Within the SPT framework of Fernholz, the solution to the drift-maximization problem at a fixed tracking-risk budget generates two-fund separation. The investor’s optimal portfolio can be constructed by holding—in a ratio depending on the desired tracking-risk level only—each of the market portfolio and a risky portfolio consisting of the linear combination of four subportfolios using rule-based, non-cap-weighted allocation schemes: a global minimum-variance portfolio, an equally weighted portfolio, a risk parity portfolio in its simpler inverse volatility variant, and a dividend-weighted portfolio.

The interest in this derivation is twofold: first, whereas in the financial literature smart beta strategies have always been considered heuristic, the SPT approach allows derivation of them from first principles. Secondly, the result shows that the portfolio exhibiting the maximum drift relative to the corresponding reference market index at a fixed tracking-error-risk budget is a combination of four elementary smart beta “seeds.”

Armed with this theoretical result, in this research note we investigate the possibility of building a “portable smart beta” program based on the smart beta index selection suggested by SPT optimization. Analogously to a “portable alpha” program, where the objective is to extract skill (“alpha” in finance community jargon) from a portfolio of actively managed investment strategies, in a “portable smart beta” program we aim to extract a combination of alternative risk premiums from the set of four smart beta indices emerging from the SPT optimization exercise.

The commonly used technique to achieve performance transport is beta hedging, which can be easily shown to minimize the volatility of a portable performance strategy. Unfortunately, beta hedging fails to control for short-term volatility. We show in this research note that the volatility of a portable smart beta strategy based on simple beta hedging can be subject to large spikes.

To resolve this problem, we propose to use the volatility targeting methodology of Giese [2010]. We define the weighting allocated to a given beta-hedged portable smart beta strategy at each point in time in such a way that a volatility target level is achieved. It can be analytically shown that any strategy with positive expected tradeoff between return and risk improves its efficiency when it is subject to a volatility-targeting rule. The empirical results we find for each of our portable smart beta strategies confirm the theoretical findings.

Portable smart beta strategies, even after volatility targeting, can be subject to large drawdowns, since in the end they are exposed to combinations of alternative risk factors, which in certain phases of the market cycle are not remunerated with a premium. Zimmermann *et al*. [2003] suggest predicting alternative risk factor performance based on a fundamental multi-factor model. This approach is difficult to implement, since it would be prone to errors both in the estimation of strategy sensitivities to alternative risk factors and in the alternative risk factor return prediction. We propose instead to use a simple trend-following rule, whose purpose is to cut exposure to each given risk-controlled portable smart beta strategy when an unfavorable trend is identified.

We show that this simple timing rule allows to significantly reduce strategy drawdowns and improve efficiency. Finally, we show that aggregating risk-controlled portable smart beta strategies based on different methodologies leads to a dramatic increase in efficiency.

The research note is organized as follows: In Section 2, we introduce the selection of smart beta indices we intend to work with, reviewing the theoretical arguments behind our choice. The present document is based on European smart beta indices, but we have checked that similar results hold also for the US and Japan.

In Section 3, we review the notion of beta hedging, introducing the volatility-targeting methodology. For every chosen portable smart beta strategy, we compare return and risk characteristics of the long/short beta-hedged solution with those of the long/short beta-hedged solution with volatility targeting, verifying the superiority of the latter approach (as predicted by theory).

In Section 4, we discuss the opportunity of timing risk-controlled portable smart beta strategies. For each strategy, we study its sensitivities to alternative risk factors, capturing size and value. First, we show that factor sensitivities are subject to significant changes over time. Timing portable smart beta strategies based on their variable sensitivities to equity alternative risk factors and on a fundamental multi-factor model would therefore seem to be a very difficult goal to achieve. We show that a simple trend-following rule improves the return-versus-risk characteristics of passively invested, risk-controlled portable smart beta strategies.

Finally, in Section 5, we analyze the properties of a portfolio that aggregates different types of risk-controlled portable smart beta strategies defined on the European equity market. We show that such a portfolio could indeed represent an interesting opportunity to enhance the performance of cash or of an arbitrary market index. We leave to a forthcoming publication the presentation of the properties of a portfolio that aggregates different types of risk-controlled portable smart beta strategies defined on three different geographies (Europe, the US and Japan). We anticipate that geographic diversification only further improves strategy efficiency, given that the behavior of alternative risk factors such as size and value depends on the evolution of the market cycle, which differs from one region or country to another.

In Section 6, we conclude by providing some hints concerning how to implement a diversified portable smart beta strategy in a specific investment vehicle.

In the document, we deliberately avoid lengthy mathematical derivations, referring the reader to the literature where necessary, and providing only the intuition behind the few formulas we quote.

2. Elementary smart beta building blocks

The elementary smart beta building blocks we use throughout this research note are derived based on the SPT framework of Fernholz [2002]. SPT provides insights into questions related to market structure and equilibrium and has been used in practice since 1990 to construct portfolios with risk-controlled behavior. It is defined on a closed universe of assets (companies are not supposed to merge and break up), whose logarithmic returns are normally distributed.

Probably the most important difference between modern portfolio theory and SPT is that, whereas modern portfolio theory has emphasized the role of expected rate of return and variance of a portfolio of assets in order to build an optimal allocation, SPT shows that it is the logarithmic growth rate both of a single asset and of a portfolio that determines their long-term behavior.

Imagine, for instance, taking a single very volatile stock whose price in the first year doubles and in the second year halves. Whereas the stock price would remain unchanged over the two-year period, the average asset simple rate of return would be 25%, which is a clear indication that the average simple rate of return is not a good predictor of the long-term performance of a volatile asset. On the contrary, the average logarithmic rate of return would be $\log 2$ in the first year and $\log \frac{1}{2}$ in the second year, averaging to zero over the two-year period.

If we call R_i the average simple rate of return of asset i over a time interval Δt and r_i the corresponding average logarithmic growth rate, the two quantities are related by Ito's rule (see, for instance, Ito [1951]):

$$R_i \, \Delta t = \left(r_i + \frac{1}{2}\sigma_i^2\right)\Delta t \tag{1}$$

where σ_i^2 is the asset variance computed on logarithmic returns. In our example, the stock variance would be equal to $(\log 2)^2$, which is approximately equal to 50%, correctly implying that the average stock simple rate of return differs from the average logarithmic rate of return by 25%.

In the logarithmic growth rate, we may want to further distinguish a contribution coming from price, $\hat{r}_i$, from a contribution coming from dividends, δ_i, and rewrite the previous equation as:

$$R_i = \left(\hat{r}_i + \delta_i + \frac{1}{2}\sigma_i^2\right) \tag{2}$$

When we move one step further and consider a portfolio in which asset i has a weighting equal to w_i, its expected simple rate of return is equal to:

$$R_p = \sum_{i=1}^{N} w_i R_i \tag{3}$$

Its expected logarithmic growth rate, r_P, can also be derived by using the analog of Ito's rule, Eq. (1)—applied to portfolio return, and then by rewriting R_P according to Equations (2) and (3):

$$r_P \Delta t = \sum_{i=1}^{N} w_i \hat{r}_i \Delta t + \sum_{i=1}^{N} w_i \delta_i \Delta t + \frac{1}{2}\left[\sum_{i=1}^{N} w_i \sigma_i^2 - \sigma_P^2\right] \Delta t \tag{4}$$

The logarithmic growth rate of a portfolio of stocks, which is the correct predictor of portfolio long-term performance, differs from the portfolio weighted average of the stocks' logarithmic growth rates by an important term (which Fernholz calls excess growth rate) and is equal to the difference between the portfolio weighted average of stock variances and the variance of the portfolio. Intuitively, the variance of a portfolio should be lower than the portfolio weighted average of its constituent variances because of diversification. It is indeed possible to prove mathematically that, for a long-only portfolio, excess growth rate is always a positive quantity.
Portfolio returns are typically compared to the returns of market-cap-weighted benchmarks, where stocks' weightings are given by $w_{BM,i}$. Fernholz [2002] shows that the relative logarithmic growth rate of a portfolio compared to a cap-weighted benchmark is given by an expression very similar to Eq. (4):

$$(r_P - r_{BM})\Delta t = \sum_{i=1}^{N}\left(w_i - w_{BM,i}\right)\delta_i \Delta t + \frac{1}{2}\left[\sum_{i=1}^{N} w_i \sigma_i^2 - \sigma_P^2\right]\Delta t \tag{5}$$

Notice that for each time interval Δt, the relative return between portfolio and benchmark consists of the above drift term, which describes the long-term relative logarithmic growth rate, and also of a stochastic term, which incorporates the random dynamics of stock prices. The stochastic term can be written as:

$$\sum_{i=1}^{N} w_i \Delta \log\left(w_{BM,i}\right) \tag{6}$$

The random dynamics of stock prices is hidden inside the dynamics of logarithmic benchmark weightings, which depend on how stock prices move relative to each other. If we work under the two main assumptions of SPT, namely that we deal with a closed asset universe in which company defaults are not possible, and that market diversity holds—implying that no single asset can take the whole capitalization of the market, the stochastic component of portfolio return relative to the benchmark in Eq. (6) remains bounded.
An investor should wish to maximize at each step in time the logarithmic growth rate of a portfolio of stocks relative to the cap-weighted benchmark in Eq. (5), while respecting the budget constraint and a tracking error risk constraint. Notice, incidentally, that maximizing the excess growth rate component alone in Eq. (5) would be a very similar problem to finding the maximum diversification portfolio of Choueifaty and Coignard [2008], since excess growth rate and the so-called "diversification ratio" of Choueifaty and Coignard are close relatives.
Oderda [2015] shows that the solution to this optimization problem can be constructed by holding—in a ratio depending on the desired tracking-risk level only—each of the market portfolios and a risky portfolio consisting of the linear combination of four subportfolios using

rule-based, smart beta allocation schemes: a global minimum-variance portfolio, an equally weighted portfolio, a risk parity portfolio in its simpler inverse-volatility variant and a dividend-weighted portfolio based on risk-adjusted dividend yield. Mathematically we have:

$$w_{OPT,i} = Aw_{BM,i} + (1-A)\left[B_{MV}w_{MV,i} + B_{EW}w_{EW,i} + B_{IV}w_{IV,i} + B_{HD}w_{HD,i}\right] \quad (7)$$

where the parameter A depends on the desired tracking-risk level, and the parameters B_J need to sum to one. The result is interesting, since it derives smart beta strategies from first principles as solutions of a long-term relative growth rate-maximization problem. Furthermore, the result shows that the portfolio exhibiting the maximum drift relative to the corresponding reference market index at a fixed tracking error risk budget is a combination of just four elementary smart beta "seeds."

In the rest of this research note, we work with the elementary smart beta building blocks identified in Oderda's work for the European equity market. Our aim is to show that, if we take a portfolio made of the diversified set of smart beta indices appearing in Eq. (7), its relative performance versus the market-cap-weighted benchmark can be used to enhance the return of cash or of any predefined benchmark. The same portfolio could also be employed as a long-only investment solution with the aim of beating the corresponding benchmark of market-cap-weighted indices. Table 1 lists the STOXX index proxies we use for each of the smart beta building blocks in Eq. (7). The four indices correspond to the elementary smart beta "seeds" derived by Oderda [2015].

TABLE 1: LIST OF EUROPEAN STOXX INDICES FOR CONSTRUCTING RISK-CONTROLLED PORTABLE SMART BETA STRATEGIES

Geography	Smart Beta Building Block	Index Proxy	Index Provider	Daily Index Coverage Start	Notes
Europe	Minimum Variance	Stoxx® Europe 600 Minimum Variance Index	STOXX	01/07/2002	The STOXX® Europe 600 Minimum Variance Index weights its constituents so that portfolio variance is minimized. STOXX uses Axioma's factor model for the optimization process. The Constrained version creates a portfolio similar to the underlying benchmark index, but with a more attractive risk profile. This is achieved by applying a range of factors, country and industry exposure constraints to ensure that components have no high allocation bias.
	Equal Weighting	Stoxx® Europe 600 Equal Weight Index	STOXX	02/01/2001	The STOXX® Europe 600 Equal Weight Index gives equivalent representation to large, mid and small capitalisation companies. With a fixed number of 600 components, the index covers 18 countries from the European region.
	Inverse Volatility	Stoxx® Europe Low Risk Weighted 300 Index	STOXX	19/03/2001	Components are selected based on a 12-month historical volatility ranking. Components are ranked from lowest to highest volatility. Weights are calculated by using the inverse of the 12-month historical volatility.
	High Dividend	Stoxx® Europe Select Dividend 30 Index	STOXX	30/12/1998	The index universe is the relevant benchmark index. Only dividend paying companies are eligible and those stocks must have a nonnegative historical five-year dividend-per-share (DPS) growth rate and a defined dividend to EPS ratio. For the selection, components are sorted by an outperformance factor to their home market and ranked accordingly. A minimum liquidity level and a buffer rule are applied. Weighting is dividend-based

Source: STOXX

With respect to the theoretical result, the indices we use are proxies. For instance, whereas the theoretical result would say the inverse-volatility portfolio needs to be built with all the universe constituents, all the low-volatility indices we use are built only with a universe subsample. Whereas the theoretical result would say the high-dividend portfolio needs to be based on risk-adjusted dividend yields, the high-dividend indices we employ do not apply any risk-adjustment on dividends to compute the weightings.

In this sense, we believe this research note cannot be considered an exact empirical check of Oderda's theoretical result.

3. Portable smart beta strategies: risk control on top of simple beta hedging

Any portable performance strategy consists of a long/short portfolio, with the short side hedging the market risk exposure of a long portfolio that has the potential to beat the benchmark. A well-known example is provided by the so-called “portable alpha” strategies, where a portfolio of long-only actively managed investment programs is bought, and benchmark exposure is hedged with futures and/or total-return swaps to extract the value added by investment professionals’ skill (“alpha” in finance community jargon).

Our aim on the long side won’t be to buy actively managed products, but rather to select long-only smart beta indices, trying to extract their long-term added value relative to the relevant market-cap-weighted benchmarks.

Whereas it is certainly true that portable performance from smart beta indices has nothing to do with investment skill or alpha (it comes from exposure to alternative risk factors), it is well known that in many cases what is sold as “portable alpha” at very high management fees is in reality (at least partly) a collection of risk premia coming from alternative risk factor exposure.

Compared to a portable alpha program, we believe a portable smart beta strategy based on a diversified selection of smart beta approaches would offer advantages in terms of transparency, liquidity and cost:

1. **Transparency**: Whereas in a portable alpha program the portfolio positions of an actively managed product are disclosed only with delay and at limited frequency (e.g., once a month), the composition of a smart beta index is immediately available.
2. **Cost**: Whereas the fees of active products managed by “star alpha generators” tend to be quite high, the fees required to get exposure to smart beta indices are very low.

   Access to smart beta indices can be obtained either in unfunded form (by means of a total-return swap) or in funded form (by directly purchasing the index’s underlying holdings or by means of exchange-traded funds [ETFs], which have been made available for many smart beta indices). Unfunded exposure to smart beta indices via total-return swaps can be achieved at very low cost (at current interest rate levels, even at zero or a slightly negative cost), if the swap counterparty can profit, for instance, from tax recovery on dividends. A great advantage of the unfunded approach is the possibility to exploit a moderate degree of leverage, if needed, and the opportunity to manage cash independently.

   Funded exposure to smart beta indices is in our opinion suboptimal, both because it is less flexible and because it has higher costs. For instance, typical smart beta ETFs may charge management fees of 40-50 basis points per annum, which are slightly higher than the swap fees incurred with the unfunded approach.

3. **Liquidity**: Whereas there might be issues with the management of flows in a portable alpha program (frequent flows affecting an actively managed fund may not be tolerable for the investment managers), the liquidity of a portable smart beta strategy should be higher, considering that nowadays many ETFs that track the performance of smart beta indices have appeared in the market.

   The standard technique to extract portable performance from a long-only portfolio is beta hedging. Suppose that our portable smart beta strategy return, R_{PSB} , comes from the returns of a long position in a smart beta index, R_{SB}, and of a short position in the corresponding cap-weighted benchmark, R_{BM} , with weighting w_{BM} :

$$R_{PSB} = R_{SB} - w_{BM} R_{BM} \tag{8}$$

   The expected variance of this strategy is given by:

$$\sigma_{PSB}^2 = \sigma_{SB}^2 + w_{BM}^2 \sigma_{BM}^2 - 2w_{BM} \sigma_{SB,BM} \tag{9}$$

   where $\sigma_{SB,BM}$ is the covariance between the smart beta index and the cap-weighted benchmark. If our objective is to minimize the variance of the long/short portfolio, it is easy to see that the hedge weighting should be equal to the ex-ante beta of the smart index relative to the cap-weighted benchmark:

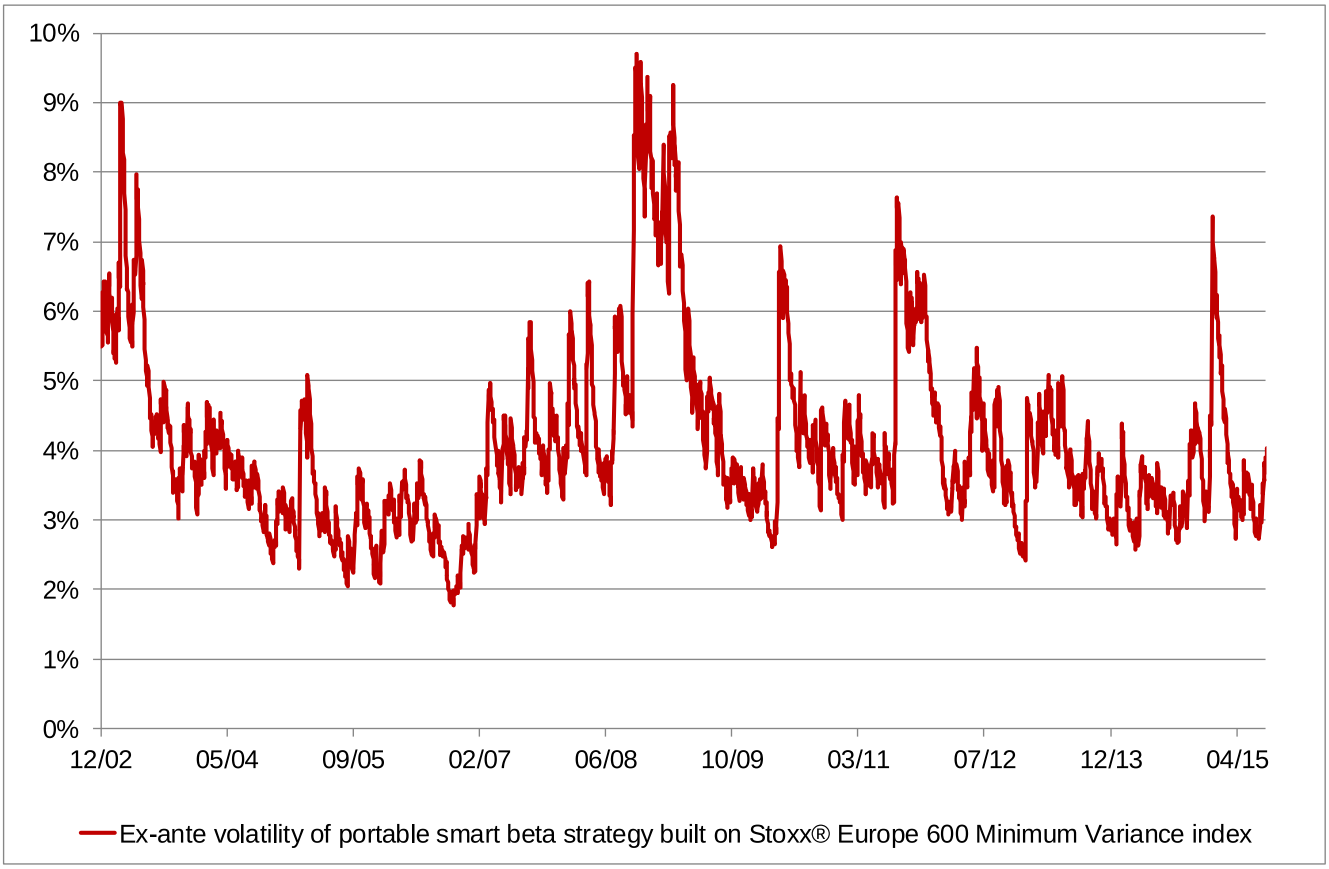
$$w_{BM} = \beta_{SB,BM} \tag{10}$$

   With this hedge weighting, the variance of the portable performance strategy is given by:

$$\sigma_{PSB}^2 = \sigma_{SB}^2 \left(1 - \rho_{SB,BM}^2\right) \tag{11}$$

   where $\rho_{SB,BM}$ is the correlation between the smart beta index and the cap-weighted benchmark. It is immediately seen that, if the correlation between the smart beta index and the cap-weighted benchmark breaks down, the variance of the portable performance strategy can dramatically increase, even though by definition it is the minimum attainable variance. This is what we observe in Figure 1, where we plot the volatility of the portable smart beta strategy built on the STOXX Europe 600 Minimum Variance Index, calculated over a daily data sample using exponentially weighted returns based on the RiskMetrics optimal decay factor, $\lambda = 0.94$.

FIGURE 1: ANNUALIZED EX-ANTE VOLATILITY OF THE PORTABLE SMART BETA STRATEGY BUILT ON THE STOXX EUROPE 600 MINIMUM VARIANCE INDEX (EUR, NET RETURNS, SEPTEMBER, 2002-NOVEMBER, 2015)



Source: STOXX, own calculations[1]

We observe that volatility is subject to oscillations, which in some cases can be quite large. To keep volatility under control, we use the methodology proposed by Giese [2010]. The idea is to define an ex-ante volatility objective, T, to measure on a daily basis the volatility of the portable smart beta strategy based on beta hedging, $\sigma_{PSB,t}$, allocating to the portable smart beta strategy a weighting equal to

$$w(\sigma_{PSB,t}) = \frac{T}{\sigma_{PSB,t}} \qquad (12)$$

In other words, we decrease strategy weighting when volatility increases, and vice versa. Giese shows that, if we consider the expected Sharpe ratio of a portable smart beta strategy subject to volatility targeting, $S_{PSB,VT}$, it differs from the expected Sharpe ratio of the original strategy, S_{PSB}, according to the following formula:

[1] Strategy volatility is calculated by means of Eq. (11), where the volatility of both the long index and the market risk hedge are estimated based on exponentially weighted daily logarithmic returns (exponential weighting parameter taken equal to the RiskMetrics optimal decay factor, λ = 0.94).

$$S_{PSB,VT} - S_{PSB} = S_{PSB}\frac{\mathrm{vol}(\sigma_{PSB})^2}{\overline{\sigma}_{PSB}} + \frac{1}{2}(\overline{\sigma}_{PSB} - T) + \frac{\mathrm{vol}(\sigma_{PSB})^2}{2\overline{\sigma}_{PSB}} \tag{13}$$

where $\overline{\sigma}_{PSB}$ is the expected volatility of the portable smart beta strategy and where $\mathrm{vol}(\sigma_{PSB})$ is the strategy volatility of volatility.

This is a very strong result, since it shows that in the long run, irrespective of the underlying strategy and irrespective of the probability distribution of volatility, the risk-controlled portable smart beta strategy should always create a better Sharpe ratio than the underlying portable smart beta strategy, as long as the underlying strategy-expected Sharpe ratio is positive. Also, the volatility target must be chosen to be smaller or equal to the expected volatility of the underlying strategy. It is also interesting to note that the improvement of the expected Sharpe ratio is driven by the volatility of the volatility, which plays the role of an "opportunity parameter." The more volatile the volatility of the portable smart beta strategy is, the higher the expected Sharpe ratio improvement of the volatility-controlled strategy would be.

In Table 2, panels (a)-(d), we show the full set of results applied to the selection of indices in Table 1. In each case, we measure beta over a window of 66 trading days (roughly equivalent to one quarter), and we calculate the volatility of the portable smart beta strategy according to Eq. (11). Panel (a) shows the statistics for the STOXX Europe 600 Minimum Variance Index (EUR, net returns), panel (b) for the STOXX® Europe Low Risk Weighted 300 Index (EUR, net returns), panel (c) for the STOXX Europe 600 Equal Weight Index (EUR, net returns) and panel (d) for the STOXX® Europe Select Dividend 30 Index (EUR, net returns)[2]. The reported return figures are in excess of cash.

[2] In each panel we distinguish three columns: the first reports return-versus-risk statistics for the version of each strategy, subject to simple beta hedging; the second reports return-versus-risk statistics for the version of each strategy, subject to beta hedging and volatility targeting; the third reports return-versus-risk statistics, subject to beta hedging, volatility targeting and market timing performed with a simple trend-following rule explained in Section 3.

TABLE 2: RETURN-VERSUS-RISK STATISTICS FOR THE PORTABLE SMART BETA STRATEGIES BUILT ON THE FOUR STOXX SMART BETA INDICES OF TABLE 1

Panel (a): Stoxx® Europe 600 Minimum Variance	Beta hedging	Beta hedging with volatility targeting at 5% p.a.	Beta hedging with volatility targeting at 5% p.a. and with timing
Data sample	Jul 2002-Oct 2015	Jul 2002-Oct 2015	Jul 2002-Oct 2015
Annualized return	5,31%	7,79%	6,49%
Annualized volatility	4,45%	5,60%	4,88%
Annualized semivolatility	2,63%	3,27%	2,98%
Skewness	-0,13	-0,16	-0,12
Kurtosis	5,18	4,71	6,96
Sharpe ratio	1,20	1,39	1,33
Sortino ratio	2,02	2,38	2,17
Omega ratio	1,22	1,25	1,29
Calmar ratio	0,31	0,48	0,43
Maximum drawdown	-16,90%	-16,23%	-14,95%
Ulcer index	5,43	5,90	5,46
Strategy annualised turnover	4,57	6,47	8,37
Improvement in Sharpe ratio p-value		0,00%	

Panel (b): Stoxx® Europe Low Risk Weighted 300	Beta hedging	Beta hedging with volatility targeting at 5% p.a.	Beta hedging with volatility targeting at 5% p.a. and with timing
Data sample	Mar 2002-Oct 2015	Mar 2002-Oct 2015	Mar 2002-Oct 2015
Annualized return	4,16%	9,71%	8,97%
Annualized volatility	3,07%	5,67%	4,95%
Annualized semivolatility	1,85%	3,33%	3,01%
Skewness	-0,33	-0,36	-0,40
Kurtosis	5,58	4,94	6,90
Sharpe ratio	1,35	1,71	1,81
Sortino ratio	2,25	2,92	2,98
Omega ratio	1,25	1,31	1,40
Calmar ratio	0,37	0,53	0,62
Maximum drawdown	-11,35%	-18,20%	-14,38%
Ulcer index	4,21	6,88	4,89
Strategy annualised turnover	4,37	10,22	12,74
Improvement in Sharpe ratio p-value		0,00%	12,14%

Panel (c): Stoxx® Europe 600 Equal Weight	Beta hedging	Beta hedging with volatility targeting at 5% p.a.	Beta hedging with volatility targeting at 5% p.a. and with timing
Data sample	Dec 1999-Oct 2015	Dec 1999-Oct 2015	Dec 1999-Oct 2015
Annualized return	2,38%	3,53%	3,72%
Annualized volatility	4,41%	5,51%	4,35%
Annualized semivolatility	2,66%	3,31%	2,76%
Skewness	0,12	-0,28	-0,33
Kurtosis	10,90	5,30	7,75
Sharpe ratio	0,54	0,64	0,85
Sortino ratio	0,90	1,07	1,35
Omega ratio	1,11	1,12	1,20
Calmar ratio	0,14	0,20	0,38
Maximum drawdown	-16,62%	-18,11%	-9,74%
Ulcer index	4,21	8,23	5,33
Strategy annualised turnover	4,45	8,14	10,81
Improvement in Sharpe ratio p-value		0,01%	0,00%

Panel (d): Stoxx® Europe Select Dividend 30	Beta hedging	Beta hedging with volatility targeting at 5% p.a.	Beta hedging with volatility targeting at 5% p.a. and with timing
Data sample	Dec 1999-Oct 2015	Dec 1999-Oct 2015	Dec 1999-Oct 2015
Annualized return	2,37%	2,40%	2,51%
Annualized volatility	7,86%	5,46%	4,13%
Annualized semivolatility	4,82%	3,13%	2,53%
Skewness	-0,20	0,12	0,17
Kurtosis	13,00	5,13	7,43
Sharpe ratio	0,30	0,44	0,61
Sortino ratio	0,49	0,77	0,99
Omega ratio	1,06	1,08	1,14
Calmar ratio	0,06	0,08	0,18
Maximum drawdown	-38,57%	-30,45%	-13,85%
Ulcer index	21,95	15,29	7,30
Strategy annualised turnover	5,23	5,03	5,65
Improvement in Sharpe ratio p-value		0,00%	0,00%

Source: STOXX, own calculations

In each panel, we show the time interval over which the relevant portable smart beta strategy has been evaluated, together with the relevant return-versus-risk statistics. Statistical measures are shown in the first column for the simple beta-hedged version of each strategy, in the second column for the passively managed risk-controlled version, and in the third column for the risk-controlled version of the strategy subject to a simple market-timing trend-following rule, to be discussed in Section 3.

We use an ex-ante volatility objective, T, equal to 5% per annum. We emphasize that the main conclusions do not depend on the choice of ex-ante volatility objective (we tried different ex-ante volatility objectives, confirming the main findings we outline in this document).

In each panel, we show a selection of efficiency measures: the Sharpe [1994] ratio measures the tradeoff between strategy excess annualized return and volatility. The Sortino [1991] ratio measures the tradeoff between strategy excess annualized return and semi-volatility, defined as volatility of negative returns only.

The Omega ratio of Keating and Shadwick [2002] measures the ratio between the area of the distribution of excess returns, respectively above and below the zero thresholds. Simply explained, an Omega ratio of one means that positive and negative excess returns are equally likely. An Omega ratio above (or below, respectively) one means that positive (or negative, respectively) excess returns are statistically more likely.

The Calmar ratio of Young [1991] measures the tradeoff between strategy excess annualized return and maximum drawdown.

Finally, the Ulcer index of Martin [1999] measures the depth and duration of price drawdowns from earlier highs. Technically, it is the square root of the mean of the squared percentage drawdowns in value. The greater the drawdown value, and the longer its recovery period to previous highs, the higher the Ulcer index.

The panels also report the annualized turnover of each strategy. For the simple beta-hedged version of the portable smart beta strategy, turnover comes entirely from hedging. For the risk-controlled version, turnover should be higher, since it can also come from the long side of the strategy. Turnover obviously has an impact on trading costs. A reasonable estimate of transaction costs can be obtained by assuming a 5-basis-point in/out fee on the total-return swap and the bid/ask spread as a percentage of mid price on the futures (a reasonable trading cost assumption would be between 3 and 5 basis points).

We report the p-value of the difference in efficiency, measured in terms of Sharpe ratio, between the simple beta-hedged version of each portable smart beta strategy and the corresponding risk-controlled version, in order to measure the statistical significance of the improvement. We use the methodology proposed in the paper by Opdyke [2007], which generalizes previous results from Lo [2002].

We see that the efficiency improvement predicted by Eq. (13) when we change from the simple beta-hedged version of each portable smart beta strategy to the corresponding risk-controlled version is verified in all cases. In fact, the p-value of the difference in efficiency is lower than 1% for each strategy. Assuming a significance level of 1%, we can state that the risk-controlled version of each portable smart beta strategy with volatility targeting at 5% per annum is significantly more efficient than the corresponding simple beta-hedged version of each strategy.

4. Timing risk-controlled portable smart beta strategies

From Table 2 it is evident that passive exposure to any risk-controlled portable smart beta strategy might imply considerable drawdowns. This finding should not come as a surprise, since we know the performance of each portable smart beta strategy is the result of exposure to alternative risk factors. It would, therefore, be preferable to time the exposure to each risk-controlled portable smart beta strategy. One way of doing that is to build a fundamental timing model capable of predicting the performance of alternative risk factors, once the mapping of each portable smart beta strategy on the different factors has been defined.

Zimmermann *et al*. [2003] identify a set of observable variables that can predict, for example, the value-versus-growth spread. The set of variables includes confidence indicators such as the US purchasing manager index and the US credit spread; growth indicators such as the GDP-weighted term spread, defined as an aggregate of long-term interest rates minus the corresponding inflation rates; liquidity indicators such as the Treasury-Eurodollar spread, which is a proxy for the current and expected health of the global financial system (and as such should be negatively related to global risk premiums) and valuation indicators such as a GDP-weighted aggregate of the dividend yields on the stock markets in the G7 countries. The model predicts the value-versus-growth spread at time t based on the reading of observables at time $t-1$.

The problem with this approach is twofold. First (and perhaps most importantly), the predictive power of fundamental timing models is limited. Secondly, the mapping of a given portable smart beta strategy on alternative risk factors is unstable. In Table 3, for each smart beta index typology, we show the evolution of factor exposure to the relevant European Fama/French size-style indices by performing a returns-based style analysis.

We use a Carhart [1997] four-factor model, where the risk factors used are a market factor (monthly return of the value-weighted European market index less the risk-free rate), a size factor (based on a long portfolio of small-cap companies and a short portfolio of large-cap companies), a value factor (based on a long portfolio of high book-to-market companies and a short portfolio of low book-to-market companies) and a momentum factor (based on a long portfolio of recent winners and a short portfolio of recent losers).

Table 3 reports, for each regression, the coefficient of determination, intercept and regression coefficients, with the related t-statistics and the corresponding statistical significance (measured by exceeding probability or p-value). Style regressions are estimated based on non-overlapping three-year windows of monthly simple returns. We also report regression results calculated over the full data sample. We have used the Newey-West ordinary least squares (OLS) regression estimation technique, which should allow control for autocorrelation and heteroscedasticity. Before running the regressions, we have confirmed stationarity of the time-series data by using the augmented Dickey Fuller test. For each risk-controlled portable smart beta strategy, corresponding to each relevant STOXX smart beta index, we show factor sensitivities to European Fama/French size-style indices. One, two and three asterisks denote coefficient significance, respectively, at the 10%, 5% and 1% level. The last column of the table shows the coefficient of determination, or adjusted R-squared, of each regression. Regressions are performed over non-overlapping three-year windows as well as over the full sample using monthly simple returns.

TABLE 3: RETURNS-BASED STYLE ANALYSIS RESULTS FOR THE RISK-CONTROLLED PORTABLE SMART BETA STRATEGIES CORRESPONDING TO THE SMART BETA INDICES SHOWN IN TABLE 1

Risk-controlled Portable Smart Beta Strategy	Time Window	Intercept	Market Factor Exposure	Size Factor Exposure	Value Factor Exposure	Momentum Factor Exposure	Coefficient of Determination
Stoxx® Europe 600 Minimum Variance	Oct 2002- Dec 2004	1,31%**	0,1311	0,2743**	-0,5845**	0,1081*	20,85%
	Dec 2004- Dec 2007	0,22%	0,1133	0,2936*	0,4531	0,0693	19,92%
	Dec 2007- Dec 2010	0,31%	0,0256	0,1750**	-0,0262	0,0148	8,78%
	Dec 2010- Dec 2013	0,11%	0,0426	0,0779	-0,2642*	0,2006**	31,21%
	Dec 2013- Oct 2015	0,37%	0,0895	0,0602	-0,4506**	0,2834**	46,44%
	Oct 2002- Oct 2015	0,49%***	0,0762***	0,1905***	-0,2208**	0,0675***	11,68%
Stoxx® Europe Low Risk Weighted 300	Oct 2002- Dec 2004	1,23%**	0,2308**	0,5447***	-0,3456	-0,0029	26,80%
	Dec 2004- Dec 2007	0,45%	-0,017	0,5084***	0,7029**	0,0521	20,97%
	Dec 2007- Dec 2010	0,62%**	0,0088	0,2610***	0,1743	-0,0643	26,20%
	Dec 2010- Dec 2013	0,43%**	0,0777**	0,2428*	-0,2560**	0,2169***	40,01%
	Dec 2013- Oct 2015	0,36%	0,1404**	0,2443	-0,7184***	0,0225	49,83%
	Oct 2002- Oct 2015	0,78%***	0,0807***	0,3045***	-0,1485*	-0,0191	11,28%
Stoxx® Europe 600 Equal Weight	Oct 2002- Dec 2004	-0,25%	0,312***	0,5862***	0,2310	-0,1668***	58,53%
	Dec 2004- Dec 2007	-0,10%	0,1522**	0,7100***	0,5379***	0,1306	66,48%
	Dec 2007- Dec 2010	0,40%**	0,0364*	0,4124***	0,2467**	-0,0120	59,85%
	Dec 2010- Dec 2013	-0,08%	0,1195***	0,688***	0,1039*	0,0828*	64,84%
	Dec 2013- Oct 2015	0,06%	0,1895***	0,7525***	-0,2121	0,0171	58,02%
	Oct 2002- Oct 2015	0,29%**	0,1171***	0,5287***	0,0619	-0,0354	46,20%
Stoxx® Europe Select Dividend 30	Oct 2002- Dec 2004	0,64%*	0,122*	0,2274*	0,0469	0,0139	13,91%
	Dec 2004- Dec 2007	0,24%	-0,1403	0,1233	0,5247**	-0,0468	10,70%
	Dec 2007- Dec 2010	-0,31%	-0,1309***	0,093	0,1426	-0,1095**	32,66%
	Dec 2010- Dec 2013	-0,14%	-0,0817*	-0,1628	0,0697	-0,0055	8,40%
	Dec 2013- Oct 2015	-0,03%	-0,0393	0,1898	-0,2711	-0,0822	7,11%
	Oct 2002- Oct 2015	0,19%*	-0,0752***	0,0702*	0,1073*	-0,0553*	5,79%

Source: STOXX, for style index data http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, own calculations

The explanatory power of the regressions is highest for the risk-controlled portable smart beta strategy built on the STOXX Europe 600 Equal Weight Index, where we see a constantly positive highly significant loading on the Fama/French European size factor.

The statistical significance of factor loadings for the risk-controlled portable smart beta strategy built on the other indices is not persistent, as shown by the sign of the factor loadings, which in some cases changes rather frequently and can even be counterintuitive.

For instance, we can see that the minimum-variance and low-risk indices in recent years seem to exhibit statistically significant negative exposure to the value factor and positive exposure to the momentum factor. This finding might actually be attributable to the fact that value strategies show mixed-to-negative performance in the most recent periods, with higher volatility patterns.

With the exception of the risk-controlled portable smart beta strategy built on the STOXX Europe 600 Equal Weight index, where we could use a fundamentals-timing model to predict return for the size-alternative risk factor and then estimate the risk-controlled portable smart beta strategy-expected return based on its sensitivity to the size factor, for the other strategies this methodology would not seem promising.

A much simpler timing methodology might be based on a trend-following model acting on the price-time series of each passive risk-controlled portable smart beta strategy. We could define, for instance, a moving average (consider for simplicity a simple moving average calculated over a rolling window corresponding to 26 weeks) for the price-time series of the passive strategy and a volatility channel around it, establishing the following trading rule: if the strategy level is above the upper threshold of the volatility channel, we should stay invested. If, on the contrary, the strategy level is below the lower threshold of the volatility channel, we should not be invested. If the strategy level is in the channel, we maintain the previous positioning. To limit trading frequency, we propose to read the timing signal on a weekly basis.

The purpose of this simple trend-following trading rule is mainly to cut exposure when prolonged drawdown periods are experienced by the passive risk-controlled strategies.

In Table 2, panels (a)-(d), comparing the excess return-versus-risk statistics shown in the second and third column of each panel, we can see that for all indices except the STOXX Europe 600 Minimum Variance, where drawdown figures are almost unchanged, there is a significant drawdown reduction when passing from the passive to the timed version of each risk-controlled portable smart beta strategy. The drawdown reduction is particularly significant for the two risk-controlled portable smart beta strategies built, respectively, on the STOXX Europe Select Dividend 30 and on the STOXX Europe 600 Equal Weight index. In these two cases, we see that the improvement in efficiency when passing from the passive to the timed version of the strategy is highly significant (the p-value of the difference in Sharpe ratio is lower than 1%). For the portable smart beta strategy built on the STOXX Europe Low Risk Weighted 300 Index, the improvement in efficiency is not statistically significant, whereas the efficiency remains virtually unchanged (or actually slightly worsens) for the strategy built on the STOXX Europe 600 Minimum Variance Index.

5. Building a diversified portfolio of portable smart beta strategies

The theory discussed in Section 1 says that it is optimal to combine portable smart beta strategies built on the four elementary smart beta "seeds" in a portfolio. Intuitively, diversification among exposures to the different alternative risk factors shown in Table 3 should lead to an efficiency increase.

In Table 4 we report the excess return-versus-risk statistics of the portfolio consisting of an equally weighted combination of the four European risk-controlled portable smart beta strategies. Notice that, since the four strategies are subject to volatility targeting at an annualized objective of 5%, the equally weighted portfolio should be very similar to a risk-weighted portfolio. In the table, the left column shows return-versus-risk statistics for the portfolio of strategies subject to beta hedging and volatility targeting. The right column shows the same statistics for the portfolio of strategies subject to beta hedging, volatility targeting and market timing via the simple trend-following rule presented in Section 3. The reported return figures are in excess of cash.

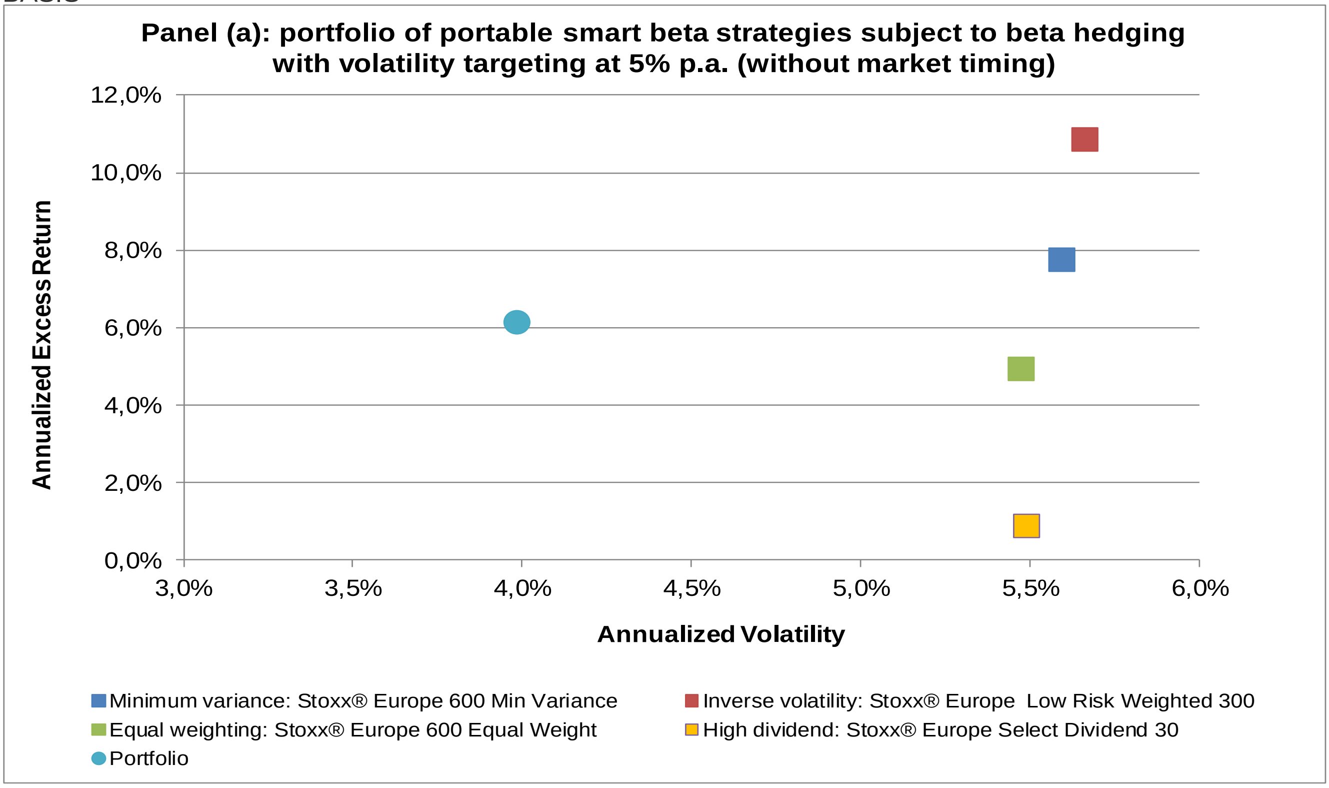
TABLE 4: RETURN-VERSUS-RISK STATISTICS OF A PORTFOLIO CONSISTING OF AN EQUALLY WEIGHTED COMBINATION OF THE FOUR EUROPEAN RISK-CONTROLLED PORTABLE SMART BETA STRATEGIES

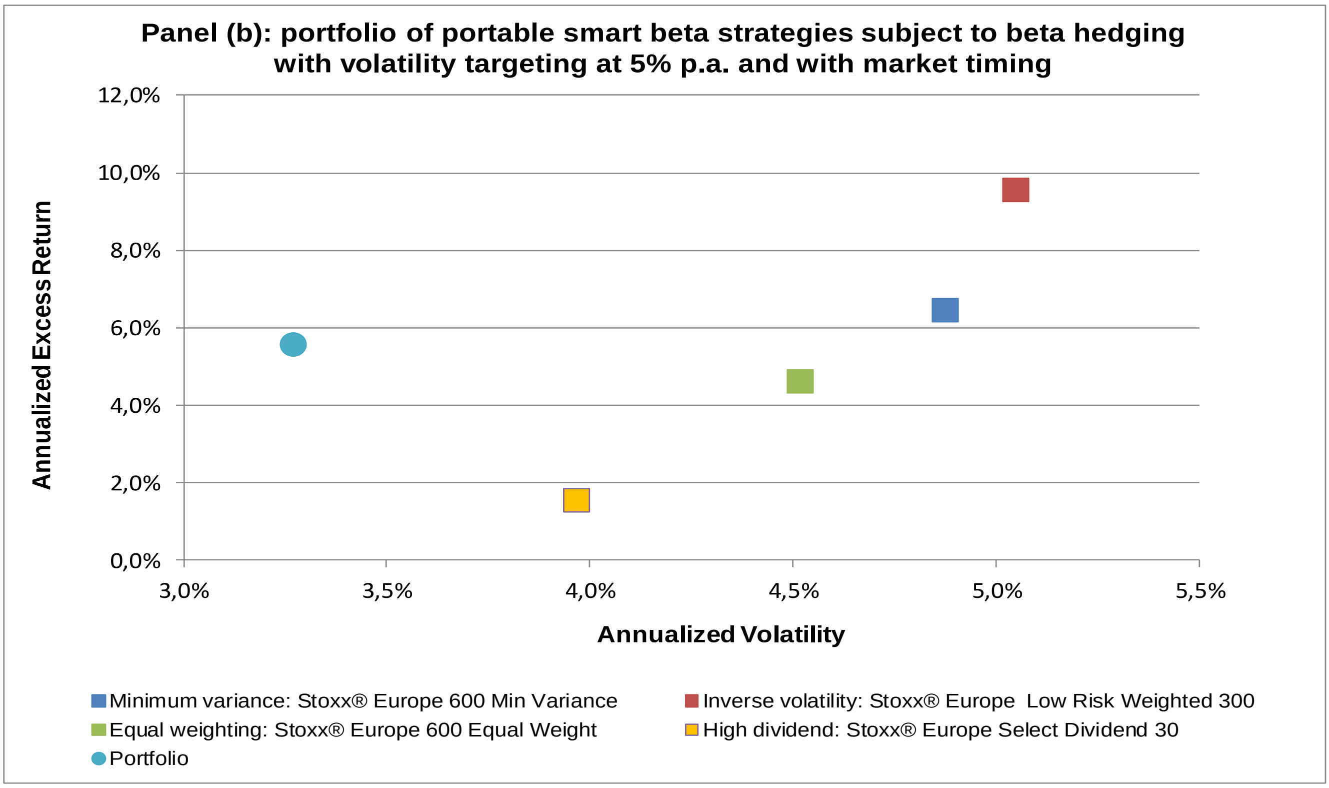
Portfolio of risk-controlled portable smart beta strategies	**Beta hedging with volatility targeting at 5% p.a.**	**Beta hedging with volatility targeting at 5% p.a. and with timing**
Data sample	Dec 2002-Oct 2015	Dec 2002-Oct 2015
Annualized return	6,14%	5,58%
Annualized volatility	3,98%	3,27%
Annualized semivolatility	2,31%	1,94%
Skewness	-0,21	-0,32
Kurtosis	4,50	6,56
Sharpe ratio	1,54	1,71
Sortino ratio	2,66	2,87
Omega ratio	1,28	1,35
Calmar ratio	0,39	0,54
Maximum drawdown	-15,85%	-10,26%
Ulcer index	6,14	3,56
Improvement in Sharpe ratio p-value		0,00

Source: STOXX, own calculations

Figure 2, panels (a)-(b), allows visual appreciation of the efficiency increase for the portfolio of strategies relative to each constituent sub-strategy taken on a standalone basis. Panel (a) shows the efficiency gain in the case of a portfolio of portable smart beta strategies subject to beta hedging with volatility targeting at 5% per annum (without market timing), whereas panel (b) shows the efficiency gain obtained in the case of a portfolio of portable smart beta strategies subject to beta hedging with volatility targeting at 5% per annum and with market timing.

FIGURE 2: EFFICIENCY INCREASE FOR THE PORTFOLIO OF RISK-CONTROLLED PORTABLE SMART BETA STRATEGIES RELATIVE TO EACH CONSTITUENT SUBSTRATEGY TAKEN ON A STANDALONE BASIS

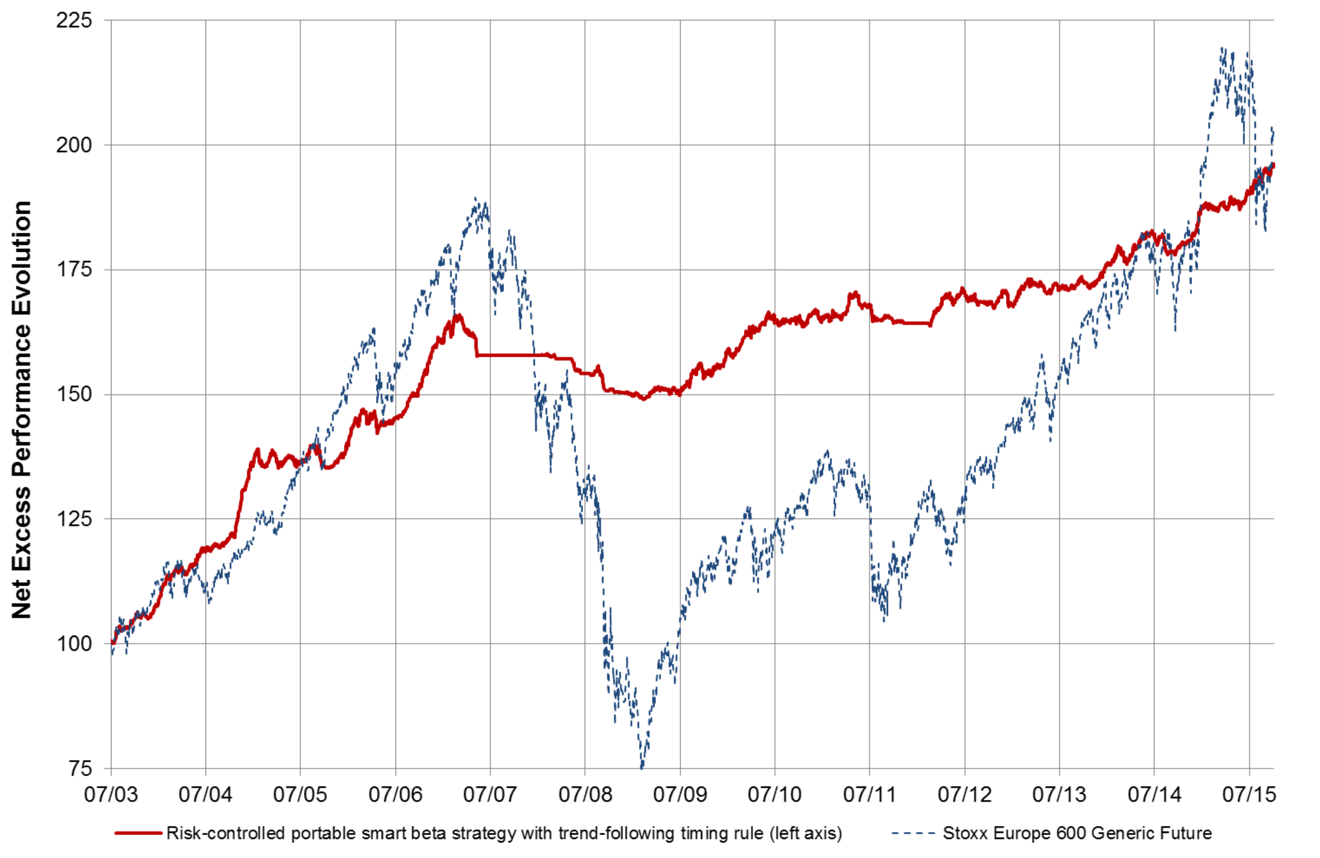




Source: STOXX, own calculations

Figure 3 shows the performance evolution in excess of cash of the portfolio of risk-controlled portable smart beta strategies subject to the trend-following timing rule introduced in Section 3. We report as a comparison the performance of the underlying market, proxied by the STOXX Europe 600 Index, to show strategy behavior in different market regimes. The excess performance evolution of the underlying equity market, proxied by the STOXX Europe 600 Index Future, is plotted for comparison reasons. Both indexed performances are defined in EUR and have been rebased at 100 on July 31, 2003. The data cover the period July, 2003-October, 2015.

FIGURE 3: EXCESS PERFORMANCE EVOLUTION OF THE PORTFOLIO OF RISK-CONTROLLED PORTABLE SMART BETA STRATEGIES, SUBJECT TO MARKET TIMING



Source: STOXX, own calculations

The results show that on a gross basis it is possible to obtain over the period 2003-2015 an annualized excess return of about 5%, with an annualized volatility of about 3%.

At Ersel Asset Management we have been implementing portable smart beta strategies since 2013 by taking unfunded exposure to the underlying indices via total-return swaps. Assuming on the four European smart beta indices actual swap fee levels of about 0.30% per annum, and transaction costs equal to 5 basis points of traded notional (whenever we increase or decrease swap exposure), we expect to achieve a net excess return of about 4% per annum. The excess return generated by the equally weighted portfolio of risk-controlled portable smart beta strategies subject to simple trend-following timing shown in Figure 3 can be used to enhance the performance of cash in absolute return investment programs or of an arbitrary benchmark replicable (at least in part) by means of derivative instruments.

6. Conclusions

In this research paper we have investigated the possibility of building a "portable smart beta" program based on a properly chosen selection of STOXX European smart beta indices. We have motivated our choice of elementary smart beta index "seeds," based on a SPT optimization exercise whose objective is to maximize the logarithmic growth rate of a portfolio of stocks relative to the corresponding cap-weighted benchmark at a fixed tracking-error risk. Our selection of smart beta index seeds includes only indices based on simple construction methodologies: a global minimum-variance portfolio, an equally weighted portfolio, a risk parity portfolio in its simpler inverse-volatility version and a dividend-weighted portfolio. For each of these methodologies, on the European equity market STOXX offers an index proxy.

We have shown that, subject to the definition of a volatility-control policy and of simple trend-following timing algorithms, whose only purpose is to protect the investor from the occurrence of large drawdowns when alternative equity risk factors (such as size, value and momentum) are not remunerated, the performance differential between a portfolio of STOXX European smart beta indices and the corresponding cap-weighted market benchmark may provide a very interesting opportunity to enhance the return of cash or of an arbitrary market index. For the period July, 2003-October, 2015, we estimate a net return enhancement of about 4% per annum, with an annualized volatility of about 3% per annum.

We may also employ the long-only portfolio of STOXX smart beta indices as an investment solution that aims to beat over time the corresponding benchmark of market-cap-weighted indices. We would simply need to adapt the logic of our trend-following timing engine in such a way that, when the signal suggests not being invested, we could replace the investment in each smart beta index with the corresponding investment in the cap-weighted index.

Recently, we have also studied the properties of a portfolio that aggregates the four types of risk-controlled portable smart beta strategies defined on three different geographies (Europe, the US and Japan). While the results of this study will be postponed to a separate publication, they fully confirm the findings outlined above. In addition, as it should be expected, geographic diversification further improves strategy efficiency, given that the behavior of alternative risk factors such as size, value and momentum depends on the phase of the economic cycle, which differs from market to market.

In concluding this research note, it is worthwhile to emphasize that access to alternative risk factors in the equity space might also be achieved by means of the long/short baskets of stocks replicating pure factors that have been made available by a few index providers and by numerous investment banks.

We believe the portable smart beta program presented in this document has the advantage of being more easily tradable and less costly, since the average fee level incurred to acquire exposure to pure-factor indices tends to be higher (in the 40-70-basis-point per annum range) to remunerate their higher degree of operational complexity.

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